$\boxed{1200}$ 面積ノ概念ニ基ヅク空間ノ幾何學(續)

岩本秀行 (名大)

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前談話ニ於テハ n 次元空間ニ於ケル K 次元曲面ノ面積ガ、積分

$$O=\int L\left(x^i, \frac{\partial x^i}{\partial u^\lambda}\right) du^1 \cdots\cdots du^K$$

デ与ヘラレル空間ノ幾何學ヲ論ジタ。ココデハ面積ヲ与ヘル積分ガ最モ一般ノ形

$$O=\int F\left(x^i, \frac{\partial x^i}{\partial u^{\lambda_1}}, \cdots, \frac{\partial^M x^i}{\partial u^{\lambda_1}\cdots\partial u^{\lambda_M}}\right) du^1\cdots du^K$$

デ与ヘラレル空間ノ幾何學化ノ問題ヲ取扱フ。F ハ勿論積分不変量ノ條件:

$$\Delta^{\lambda(S)}_{\mu} F = \sum_{(\geq)} \binom{r}{s} F_{;i}^{\lambda(S)\mu(r-S)}\, p^i_{\mu(r-S)\mu} = 0 \quad (S\geq 2)$$

$$\Delta^\lambda_\mu F = \sum_{r\geq1} r F_{;i}^{\lambda\mu(r-1)}\, p^i_{\mu(r-1)\mu} = \delta^\lambda_\mu F$$

ヲ満足シナケレバナラナイ。

§1 $\epsilon^{\lambda(M)}_i = F^{-1}F_{;i}^{\lambda(M)}$、 $\epsilon^{\lambda(M)\ \mu(M)}_{i\ \ \ \ j} = F^{-1}F_{;i\ \ j}^{\lambda(M)\mu(M)}$

ハ i, j ニ関シ共変ベクトル, λ, μ ニ関シ反変ノ Tensor デアル。

$$\epsilon^{\lambda_1\cdots\lambda_N\ \mu_1\cdots\mu_N}_{i_1\cdots i_{n-K},\ j_1\cdots j_{n-K}} = \sum_{\substack{(\lambda_1\cdots\lambda_N)\\(\mu_1\cdots\mu_N)}} \sum'_{\substack{(i_1\cdots i_{n-K})\\(j_1\cdots j_{n-K})}} \epsilon^{\lambda_1\cdots\lambda_M\ \mu_1\cdots\mu_M}_{(i_1\ \ \ j_1)} \cdots \epsilon^{\lambda_{N-M+1}\cdots\lambda_N\ \mu_{N-M+1}\cdots\mu_N}_{(i_{n-K}\ \ \ j_{n-K})} \qquad (N=(n-K)M)$$

$$u_{i_1 \cdots i_{n-k}} = \varepsilon_{i_1 \cdots i_{n-k}\, i_{n-k+1} \cdots i_n}\, p^{i_{n-k+1}}_{1} \cdots p^{i_n}_{k}$$

トナル量ヲツクレバ

$$e^{\lambda_1 \cdots \lambda_N\ \mu_1 \cdots \mu_N}_{i_1 \cdots i_{n-k} \cdot j_1 \cdots j_{n-k}} = u_{i_1 \cdots i_{n-k}} \cdot u_{j_1 \cdots j_{n-k}}\, G^{\lambda_1 \cdots \lambda_N\, \mu_1 \cdots \mu_N}$$

ナル如キ量 $G^{\lambda_1 \cdots \lambda_N\ \mu_1 \cdots \mu_N}$ ヲ求メルコトガ出来ル。ソノ行列式ガ0デナイトスレバ

$$g G^{\lambda_1 \cdots \lambda_N\ \mu_1 \cdots \mu_N} = g^{\lambda_1 \cdots \lambda_N,\ \mu_1 \cdots \mu_N} \qquad |g^{\lambda_1 \cdots \lambda_N,\, \mu_1 \cdots \mu_N}| =$$

ナル如キ座標変換デ2荷ノ intrinsic ナ scalar g ト座標変換デ不変ナ V-tensor $g^{\lambda_1 \cdots \lambda_N \cdot \mu_1 \cdots \mu_N}$ ヲ求メルコトガ出来ル。之ヲ用ヒテ次ノ拡張サレタ Christoffel ノ記号

$$G^{\lambda_1 \cdots \lambda_N}_{\mu_1 \cdots \mu_N, \nu} = \frac{1}{2} g^{\lambda_1 \cdots \lambda_N, \nu_1 \cdots \nu_N} \sum_{\alpha} \left(g_{\nu_1 \cdots \nu_N, \mu_1 \cdots \mu_N / \nu} + g_{\nu_1 \cdots \nu_N,\, \mu_1 \cdots \mu_{\alpha-1} \nu\, \mu_{\alpha+1} \cdots / \mu_\alpha} - g_{\nu_1 \cdots \nu_{\alpha-1} \nu\, \nu_{\alpha+1} \cdots \nu_N,\, \mu_1 \cdots \mu_N / \nu_\alpha} \right)$$

ヲツクリ

$$G^{\lambda}_{\mu\nu} = A\, G^{\lambda_1 \cdots \lambda_{N-1} \lambda}_{\lambda_1 \cdots \lambda_{N-1}\, \mu, \nu} + B\, \delta^{\lambda}_{\mu}\, G^{\lambda_1 \cdots \lambda_N}_{\lambda_1 \cdots \lambda_N, \nu}$$

（茲ニ A, B ハ適当ナ有理数トスル）。

トオケバ，$G^{\lambda}_{\mu\nu}$ ハ V-tensor ニ対スル擬似接続ヲ定義スル。之ハ最高次ノ面素 $p\lambda^{i}_{(M+1)}$ ニ関シテ一次デアル。

今 $e^{\lambda(M)\ \mu(M)}_{i\ \ j}$ ガ次ノ如キ性質ヲモツト假定スル。

$$f^{k}_{\nu(M)}{}^{i}_{\lambda(M)}\, e^{\lambda(M)\ \mu(M)}_{i\ \ j} = Q^{k}_{j}\, \delta^{\mu_1}_{\nu_1} \cdots \delta^{\mu_M}_{\nu_M}$$

$$Q^{k}_{j} = \delta^{k}_{j} - P^{\nu}_{j} P^{k}_{\nu}$$

ナル如キM次ノ量 $f_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{j}$ ガ少クトモ一組適当ナ P^{ν}_{j} ニ対シテ存在スル。

上ノ様ナ量ガ存在スレバ，ソレハ無数ニ存在シ，且ソノ任意

ノ二ツノ間ニハ

$$F_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{j} = f_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{j} + \varphi_{\lambda(M)}{}^{v}{}_{\mu(M)}{}^{j}\, p_v^i + \varphi_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{v}\, p_v^j$$

ナル関係ガ存在シ，逆モ亦成立スル．又 $\ell^{\lambda(M)}_{i}{}^{\mu(M)}_{j}\varphi^{j}_{\mu(M)}=0$
ナル $\varphi^{j}_{\mu(M)}$ ハ必ズ $\varphi^{v}_{\mu(M)}\,p_v^j$ ナル形トナル，又 $\Phi^{\lambda(M)}_{i}$

$\Psi^{\lambda(M)}_{i}$ ガ $\Phi^{\lambda(M)}_{i}\,p^{i}_{u} = \Psi^{\lambda(M)}_{i}\,p^{i}_{u} = 0$

ヲ満足スル不変量ナルトキ

$$f_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{j}\,\Phi^{\lambda(M)}_{i}\,\Psi^{\mu(M)}_{j}$$

ハ上ノ関係ヲ満足スル $f_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{j}$ ニ関係シナイ invariant デアル．

§2　v^i ヲ任意ノ intrinsic ナ反変 vector トスレバ

$$D_\mu(F_j{}^{\lambda(M)}_{i}, v) = F^{-1}\Big[F_{i}{}^{\lambda(M)}_{i}{}_{;}{}^{\mu(M-1)\mu}_{j}\, v^j{}_{/\mu} + \big(F_{i}{}^{\lambda(M)}_{i}{}_{;}{}^{\mu(M-1)}_{j}$$

$$- \delta^{\mu_1}_{(v_1}\cdots\delta^{\mu_{M-2}}_{v_{M-2}}\, G^{\mu_{M-1}}_{v_{M-2}v_M)}\, F_{j}{}^{\lambda(M)}_{i}{}_{;}{}^{v(M)}_{j}\big)\, v^j\Big]$$

ハ intrinsic ナ共変 vector デ μ ニ対シ U-共変 vector デアル。今

$$\nabla_\mu F_{;}{}^{\lambda(M)}_{i} = F_{;}{}^{\lambda(M)}_{i}{}_{/\mu} - \frac{1}{pF}\Big[t^{h}_{\mu(M-1)\mu}{}^{k}_{v(M)}\big(F^{v(M)\mu(M-1)}_{jh\,;\,i} -$$

$$\delta^{\mu_1}_{(w_1}\cdots\delta^{\mu_{M-2}}_{w_{M-2}}\,G^{\mu_{M-1}}_{w_{M-1}w_M)}\,F_{;}{}^{v(M)}_{h}{}_{;}{}^{w(M)}_{j}\big)F_{;}{}^{\lambda(M)}_{k}\Big]$$

$$+ \big(\sum_\alpha \delta^{(\lambda_1}_{\mu_1}\cdots G^{\lambda\alpha}_{\mu\alpha\mu}\ \delta^{\lambda_M)}_{\mu_M}\big) - \delta^{(\lambda_1}_{\mu_1}\cdots\delta^{\lambda_M)}_{\mu_M}G^{\alpha}_{\alpha\mu}\big)F^{\mu(M)}_{i}$$

$$(M \geqq 3)$$

$$\nabla_\mu F_{;}{}^{\lambda_1\lambda_2}_{i} = F_{;}{}^{\lambda_1\lambda_2}_{i}{}_{/\mu} - \frac{1}{F}\Big[t^{h}_{u\pi}{}^{k}_{v_1v_2}\big(F_{;}{}^{v_1v_2}_{h}{}_{;}{}^{\pi}_{i} - G^{\pi}_{w_1w_2}F_{;}{}^{v_1v_2}_{k}{}_{;}$$

$$\left.{}_{j}^{w_1 w_2}\right) - f_{\mu\pi}{}^{k}{}_{\nu_1\nu_2}{}^{k} E^{\pi}_{k} F;{}^{\nu_1\nu_2}_{i} \Big]$$

$$- G^{\alpha}_{\alpha\mu} F;{}^{\lambda_1\lambda_2}_{i} + 2 G^{(\lambda_1}_{\lambda\mu} F;{}^{\lambda_2)}_{i} \qquad (M=2)$$

茲ニ $E^{\lambda}_{i} = 2F;{}^{\lambda\mu}_{i}/\mu - F;{}^{\lambda}_{i} + \Gamma^{\lambda}_{\mu\nu} F;{}^{\mu\nu}_{i}$

ハ共変 vectorデ且 U-vector，又

$$p^{i}_{\nu} \nabla_{\mu} F;{}^{\lambda(M)}_{i} = 0$$

ヲ満足シ，且 $f_{\lambda}{}^{i}_{(M)}{}_{\mu}{}^{j}_{(M)}$ ノトリ方ニ無関係デアル。

又 $f_{(\lambda}{}^{i}_{(M)}{}_{|\mu}{}^{j}_{(M)|}\nabla_{\lambda)} F;{}^{\mu(M)}_{i}$

$$= (\delta^{i}_{k} - p^{\nu}_{k} p^{i}_{\nu}) \Big[(p^{k}_{\lambda(M)\lambda} + C^{k}_{(\lambda(M)\lambda)}{}^{\mu(M+1)}_{j} p^{j}_{\mu(M+1)})$$

$$+ M^{k}_{\lambda(M)\lambda}(\chi^{i}, p^{i}_{\lambda}, \cdots, p^{i}_{\lambda(M)}) \Big]$$

ナル関係が成立スル。従ツテ行列

$$\left\| K^{i}_{\lambda(M+1)}{}^{\mu(M+1)}_{j} \right\| = \left\| \delta^{k}_{j} \delta^{(\mu_1}_{\lambda_1} \cdots \delta^{\mu_{M+1})}_{\lambda_{M+1}} + C^{k}_{\lambda(M+1)}{}^{\mu(M+1)}_{j} \right\|$$

が non-singular ダトスレバ之カラ tensor

$$T^{i}_{\lambda(M+1)} = (\delta^{i}_{j} - p^{\nu}_{j} p^{i}_{\nu})(p^{j}_{\lambda(M+1)} + H^{j}_{\lambda(M+1)})$$

ヲ次ノ関係が成立スル様ニ定メルコトが出来ル。

$$f_{(\lambda}{}^{i}_{(M)}{}_{|\mu}{}^{j}_{(M)|}\nabla_{\lambda)} F;{}^{\mu(M)}_{j} = (\delta^{i}_{j} - p^{\nu}_{j} p^{i}_{\nu}) K^{j}_{\lambda(M+1)}{}^{\mu(M+1)}_{k} T^{k}_{\mu(M+1)}$$

今 Φ ヲ任意ノ U-tensor トスレバ

$$\Phi_{\|\lambda} = \Phi_{,\lambda} - \Phi^{\lambda(M)}_{i} T^{i}_{\lambda(M)\lambda}$$

ハ径数変換デ $\Phi_{,\lambda}$ ト同ジ変換ヲスル量デ $f_{\lambda}{}^{i}_{(M)}{}_{\mu}{}^{j}_{(M)}$ ニ無

関係ニキマリ、且次数Mデアル。$\Phi_{\|\lambda_1\|\lambda_2\|\cdots\|\lambda_m} = \Phi_{\|\lambda_1\cdots\lambda_m} = \Phi_{\|\lambda(m)}$ ナル記号ヲ導入スル。

§3 Fカラ次ノ所謂 *Synge* ノ *vector* ヲ導クコトガ出来ル。

$$F.E_i^{\lambda(s)} = \sum_{r\geqq s} (-1)^r \binom{r}{s} F_i^{((\lambda)(s)\lambda(r-s))}{}_{\|\lambda(r-s)}$$

$E_i^{\lambda(s)}$ ハ径数変換デソレ等ノ間デ *linear* ニ変換サレ、径数ハ $\dfrac{\partial \bar{u}^\lambda}{\partial u^{\kappa}}$ $\dfrac{\partial^2 \bar{u}^\lambda}{\partial u^\kappa \partial u^\beta}$ --- ノ *Polynomial* デアルカラ、$\Gamma^{i}_{\kappa\nu}$ 及ビソノ微分デ *intrinsic* ニスル事ガ出来ル。

但シ $\Gamma^i_{\kappa\nu}$ ハ $G^i_{\kappa\nu}$ ノ M+1 次ノ面素ヲ $-(\delta^i_j - P^\nu_j p^i_\nu)H^j_{\lambda(M+1)}$ デオキカヘタモノトスル。之ヲ $\mathfrak{e}_i^{\lambda(s)}$ トスル。

$$\mathfrak{e}_i^{\lambda(s)}\, p^i_\nu = 0 \qquad (s \geqq 2)$$

$$\mathfrak{e}^\lambda_i\, p^i_\nu = \delta^\lambda_\nu$$

今 $\mathfrak{e}_\lambda{}^i_{(M)}\, u^j_{(M)} = (\delta^i_k - \mathfrak{e}^\nu_k p^i_\nu)(\delta^j_k - \mathfrak{e}^\nu_k p^j_\nu) f_{\lambda}{}^k_{(M)}\, u^k_{(M)}$

トオケバ之ハ $f_\lambda{}^i_{(M)}\, u^j_{(M)}$ ニ無関係デアル。

$$\Lambda^i_{j\mu} = \frac{1}{pF}\, \mathfrak{e}^i_{\mu(M-1)\mu\lambda(M)}{}^k \Big(F_{;k;j}^{\lambda(M)\;\mu(M-1)}$$

$$- \sum \delta^{\mu_1}_{(\omega_1}\, \Gamma^{\mu_{M-1}}_{\omega_{M-1}\omega_M)}\; F_{;k;j}^{\lambda(M)\;\mu(M)} \Big)$$

ナル量ヲ用ヒ任意ノ反変 *vector* v^i ヨリ

$$(\delta^i_j - \mathfrak{e}^\nu_j p^i_\nu)\, v^j{}_{|\mu} + \Lambda^i_{j\mu} v^j$$

ナル *invariant* ヲ導クコトガ出来ル。之ト

$$(\mathfrak{e}^\nu_j v^j)_{;\mu} = \mathfrak{e}^\nu_j v^j{}_{|\mu} + p^i_\nu (\mathfrak{e}^\nu_{j\|\lambda} + \Gamma^\nu_{\mu\lambda} \mathfrak{e}^\mu_j)$$

ナル U-*vector* ヨリ

$$v^i{}_{;\lambda} = \frac{\partial v^i}{\partial u^\lambda} + \Gamma^i_{j\lambda} v^j,$$

$$\Gamma^{i}_{j\lambda} = \Lambda^{i}_{j\lambda} + p^{i}_{\nu}\left(\ell^{\nu}_{j\mu\lambda} + \Gamma^{\nu}_{\mu\lambda} \ell^{\mu}_{j} \right)$$

ナル曲面ニ沿フ共変微分ガ定義サレル.

$$\sum_{\gamma\geq 0} (\gamma+1)\, \Gamma^{i}_{j\lambda\, j\, k}{}^{(\mu(\gamma)\lambda)}\, dp^{k}_{\mu(\gamma)}$$

ヲ補正シテ共変微分

$$\delta v^{i} = dv^{i} + \sum_{\gamma=0}^{M-1} \Gamma^{i}_{j}{}^{\mu(\gamma)}_{k}\, dp^{k}_{\mu(\gamma)}\, v^{j}$$

ヲ得ル.

$$\ell^{k}_{\mu(M)}{}^{i}_{\lambda(M)}\, \delta F_{j}{}^{\lambda(M)}_{i}$$

$$\ell^{k}_{\mu(M)}{}^{i}_{\lambda(M)} \sum_{1\geq s} \binom{\gamma}{s} F_{j}{}^{\lambda(M)}_{i}{}^{\mu(\gamma-s),\mu(s)}_{j\, j}\, dp^{j}_{\mu(\gamma-s)}$$

ヲ補正シテ基接続

$$\begin{cases} \delta p^{i}_{\lambda(M)} = \left(\delta^{i}_{j} - \ell^{\nu}_{j} p^{i}_{\nu} \right) dp^{j}_{\lambda(M)} + \sum_{\gamma=0}^{M-1} \Lambda^{i}_{\lambda(M)}{}^{\mu(\gamma)}_{j}\, dp^{j}_{\mu(\gamma)} \\ --- \\ \delta p^{i}_{\lambda} = \left(\delta^{i}_{j} - \ell^{\nu}_{j} p^{i}_{\nu} \right) dp^{j}_{\lambda} + \Lambda^{i}_{j\lambda}\, dx^{j} \end{cases}$$

U-tensor ノ共変微分ハ

$$\delta v^{\lambda} = p^{\lambda}_{i}\, \delta\left(p^{i}_{\mu} v^{\mu} \right)$$

即チ
$$\delta v^{\lambda} = dv^{\lambda} + p^{\lambda}_{i}\left(dp^{i}_{\mu} + dp^{j}_{\mu} \sum_{\gamma=0}^{M-1} \Gamma^{i}_{j}{}^{\nu(\gamma)}_{k}\, dp^{k}_{\nu(\gamma)} \right) v^{\mu}$$

ニヨリ定義スレバヨイ.

§4.

今 $M \geq 3$ トシ $a_{\alpha} = \ell^{i}_{\alpha\beta(M-1)}{}^{j}_{\gamma(M)}\, \ell_{i}{}^{\beta(M-1)} \ell_{j}{}^{\gamma(M)}$

ナル U-vector ヲ考ヘル. $M=2$ ナラバ $\ell_{i}{}^{\beta(M-1)}$ ノ代リニ
$$\ell = \ell^{i}_{\lambda(M)}{}^{j}_{\mu(M)}\, \ell_{i}{}^{\lambda(M)} \ell_{j}{}^{\mu(M)}$$

ヲ用ヒテツクツタ Synge ノ M-1-vector ヲ用ヒテ

U-vector a_{α} ヲ導イテオク.

$$a = g^{\lambda_1\cdots\lambda_N\cdot\mu_1\cdots\mu_N}\, a_{\lambda_1}\cdots a_{\lambda_N} a_{\mu_1}\cdots a_{\mu_N}$$

$$a^{\lambda} = g^{\lambda\lambda_2\cdots\lambda_N,\,\mu_1\cdots\mu_N}\, a_{\lambda_2}\cdots a_{\lambda_N} a_{\mu_1}\cdots a_{\mu_N}$$

$$a^{\lambda\mu} = g^{\lambda\lambda_2\cdots\lambda_N.\,\mu\mu_2\cdots\mu_N}\, a_{\lambda_2}\cdots a_{\lambda_N} a_{\mu_2}\cdots a_{\mu_N}$$

トオク。

今 $g^{\lambda\mu} = p\,(\,Na\cdot a^{\lambda\mu} - (N-1)\,a^{\lambda}a^{\mu}\,)$

ノ行列式ガ０デナイトシ

$$\pm|g^{\lambda\mu}| = L^{-2}$$

ニヨリ p ヲ定メル。次ニ

$$g''_{ij} = \sigma\cdot g_{\lambda_1\mu_1}\cdots g_{\lambda_M\mu_M}\, e_i^{\lambda(M)}\, e_j^{\mu(M)}$$

トオキ $\pm g''_{[i_1|j_1|}\cdots g''_{i_{n-k}]j_{n-k}} = v_{i_1\cdots i_{n-k}} v_{j_1\cdots j_{n-k}}$

ニヨリ σ ヲ定メル。然ルトキハ

$$g_{ij} = g''_{ij} + e_i^{\nu} e_j^{\lambda} g_{\nu\lambda}$$

ニヨリ基本 tensor g_{ij} ガ定義サレ

$g_{\lambda\mu} = p_{\lambda}^{i} p_{\mu}^{j} g_{ij}$, $\quad e_i^{\lambda} = g_{ij}\, g^{\lambda\mu} p_{\mu}^{j}$ ガ成立スル。

之ヲ用ヒテ Euclid 接続ヲ導入スルコトガ出来ル。

$$\delta v^i = d v^i + \omega^i_j(d)\, v^j$$

$$\omega^i_j(d) = \sum_{\gamma=0}^{M} \gamma^{i\ \lambda(\gamma)}_{j\ k}\, dp^{k}_{\lambda(\gamma)}$$

$$\gamma^{i\ \lambda(\gamma)}_{j\ k} = \frac{1}{2} g^{ih}\, g^{*}_{hj;k}{}^{\lambda(\gamma)} \qquad (\gamma \geqq 1)$$

$$\gamma^{i}_{jk} = \frac{1}{2} g^{ih} \{ g^{*}_{hj,k} + g^{*}_{hk,j} - g^{*}_{jk,h} \}$$

茲ニ

$$dg_{ij} = \sum_{\gamma=0}^{M} g^{*}_{ij,k}{}^{\lambda(\gamma)}\, \delta p^{k}_{\lambda(\gamma)}$$

トスル。